1. **(a)** Graph is transformed three units left from basic graph $y=\sqrt{x}$, so **domain is $x \geq -3$**. Graph is vertically reflected (opens down) and is transformed 5 units up, so **range is $y \leq 5$**.

(b) For y-intercept set $y=0$…

$y=-2\sqrt{0+3}+5$

$\mathbf{y=-2\sqrt{3}+5}$

For x-intercept set $y=0$…

$0=-2\sqrt{x+3}+5$

$2\sqrt{x+3}=5$ ➡ $\sqrt{x+3}=5/2$

➡ $x+3=25/4$ *square both sides*

➡ $\mathbf{y=13/4}$ **or 3.25**

2. The domain tells us the graph is shifted 2 right, and the range tells us it's 3 down. So $h=-2$ and $k=-3$ which gives $y=a\sqrt{x+2}-3$.

Now, use the point $(-1,0)$ *(given by the x-intercept)* to solve for "a"

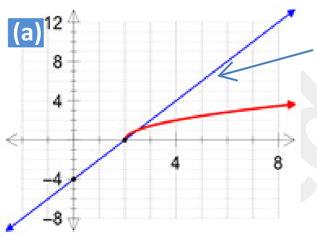
$0=a\sqrt{(-1)+2}-3$ ➡ $3=a\sqrt{1}$ $\quad a=3$ $\quad$ **ANSWER: 3**

3. For domain of a radical function, we recall that we can only sq root positive (or zero) values. So set whatever's under the sq root sign ≥ 0 and solve.

$bx+6 \geq 0$ ➡ $bx \geq -6$ ➡ $x \geq -\dfrac{6}{b}$

ANSWER: C

4. **(a)**



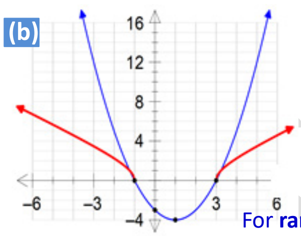
Domain of $y=\sqrt{f(x)}$ is defined by where $f(x) \geq 0$, that is, where the graph is ***above the x-axis.*** $\mathbf{x \geq -2}$

For **range**, we consider the smallest value of x we can sq root (0), and the fact that the value of $f(x)$ ***increases without bound***.

$\mathbf{y \geq 0}$

For the **equation**, of $f(x)$, we consider that it is a line with a y-intercept of -4 and a slope of 2. (We rise 4 and run 2 to get from the y to the x-intercept. So $f(x)=2x-4$ and $\quad \mathbf{y=\sqrt{2x-4}}$

(b)



For **domain** we note that $f(x)$ is below the x-axis between its x-intercepts, that is between -1 and 3. So domain of $y=\sqrt{f(x)}$ is

$\mathbf{x \leq -1}$ or $\mathbf{x \geq 3}$

For **range**, considerations are the same as before. $\quad \mathbf{y \geq 0}$

For the **equation**, of $f(x)$, we consider that it is a parabola with zeros at $x=-1$ and $x=3$. So $f(x)=a(x+1)(x-3)$ and if we use a point like (0, -3) to solve for a…

$-3=a(0+1)(0-3)$

$-3=-3a$ ➡ $\quad a=1$

$$\mathbf{y=\sqrt{(x+1)(x-3)}}$$

(c) For invariant points we consider that the numbers 0 and 1 do not change when we sq root them. So, we solve for where $f(x)=0$ and $f(x)=1$.

For (a)… $\quad 2x-4=0$ *and* $\quad 2x-4=1$

$2x=4 \qquad 2x=5$

$x=2 \qquad x=5/2$

➡ Pts are $\mathbf{(2,0)}$ and $\mathbf{(\tfrac{5}{2},1)}$

For (b)… $\quad (x+1)(x-3)=0$ *and* $\quad (x+1)(x-3)=1$

$x=-1 \quad x=3$ *(easy!)* $\qquad x^2-2x-3=1$

$x^2-2x-4=0$ ⬅Use Quad Formula!

So here we get inv. pts. $(-1,0)$ and $(3,0)$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{20}}{2} \quad ➡ x=1\pm\sqrt{5}$$

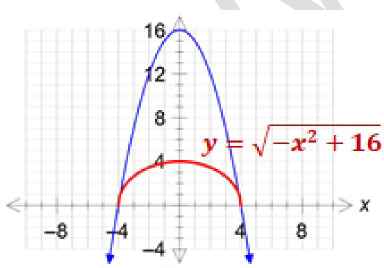
And here we get inv. pts. $(1-\sqrt{5},1)$ and $(1+\sqrt{5},1)$

So there are FOUR inv pts in total!

Verified by the graph, note how many times $f(x)$ would intersect imaginary lines at $y=0$ or $y=1$.

➡ Pts are $\mathbf{(-1,0)}$, $\mathbf{(3,\ 0)}$, $\mathbf{(1-\sqrt{5},\ 1)}$ and $\mathbf{(1+\sqrt{5},\ 1)}$

5.



DOMAIN of $y=\sqrt{f(x)}$ is defined by where $f(x) \geq 0$, that is, where the graph is ***above the x-axis.***

➡ $\mathbf{-4 \leq x \leq 4}$ **or** $\mathbf{[-4,4]}$ *Interval notation*

For **RANGE** of $y=\sqrt{f(x)}$ is defined by the min value of $f(x)$ we can sq root (that is, 0), and the max value (16).

➡ $\mathbf{0 \leq y \leq 4}$ **or** $\mathbf{[0,4]}$ *Interval notation*

For **INV. PTS we set** $f(x)$equal to the two values that have themselves as their own sq roots…..

$-x^2+16=0$ *and* $\quad -x^2+16=1$

$16=x^2 \qquad 15=x^2$

$x=\pm 4 \qquad x=\pm\sqrt{15}$

➡ Pts are $\mathbf{(-4,0)}$, $\mathbf{(4,0)}$, $\mathbf{(-\sqrt{15},1)}$ and $\mathbf{(\sqrt{15},1)}$ *four pts total*

6. **Think:** *Where is $f(x)$ above the x-axis?*

To the right of the x-intercept, which appears as some pos value.

➡ $x \geq x$-int

$x \geq$ **some pos value**

So only option is … $\quad \mathbf{x \geq 3}$

CODE: 3

ANSWER: 31

Now think: *Where is $g(x)$ above (or on) the x-axis?* It always is! (Graph never goes below)

So option is … $\quad \mathbf{x \in \mathbb{R}}$ $\quad$ **CODE: 1**

7. *Neither graph has a max. Lowest value of $f(x)$ (that can be sq rooted) is 0.*

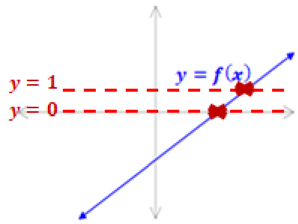
➡ $\mathbf{y \geq 0}$ $\quad$ **CODE: 2**

Lowest value of $g(x)+4$ (that can be sq rooted) is 4. $\quad \mathbf{y \geq 2}$ $\quad$ **CODE: 4**

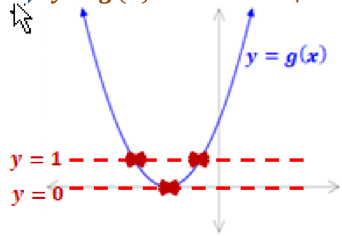
ANSWER 24

8. For invariant points we consider where the value (y-coord) of the graph is 0 or 1.

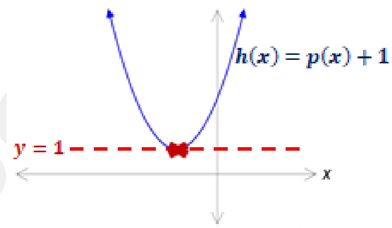
➡ $y = f(x)$ has **two** inv pts



➡ $y = g(x)$ has **three** inv pts



➡ $y = h(x)$ has **one** inv pts



ANSWER: 231

9.

$$y = \frac{2x - 5}{x + 1}$$

x-int at x value that makes the "top" 0* ➔ (c) $x = 5/2$

y-int set $x = 0$

$$y = \frac{2(0) - 5}{(0) + 1}$$

➔ $y = -5$

V.A. at x value that makes the "bottom" is 0* ➔ (a) $x = -1$

Here it's same degree top/ bottom, so H.A. at y =*ratio of lead coefficients* ➔ (b) $y = 2$

*Unless the x value makes the top **and** bottom 0, then it's a **P**oint of **D**iscontinuity. **(PD)**

10(a) V.A. at x value that makes the "bottom" is 0*

$$f(x) = \frac{5}{(x - 4)(x + 1)}$$

➔ **V.As at $x = -1$, $x = 4$**

Here it's higher degree on bottom, so H.A. at $y = 0$

➔ **H.A. at $y = 0$**

10(b) V.A. at x value that makes the "bottom" is 0*

$$f(x) = \frac{2x^2}{x(x - 3)}$$

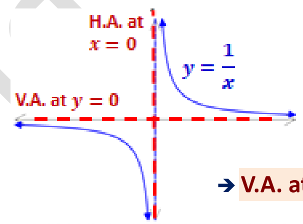
➔ **V.As at $x = 0$, $x = 3$**

Here it's same degree top/ bottom, so H.A. at y =*ratio of lead coefficients*

➔ **H.A. at $y = 2$**

10(c) For rational function in this form, think in terms of *transformations*.

From this...



To this...

$$f(x) = \frac{3}{x + 1} - 2$$

Stretch / no effect on asymptotes

Shift 1 left (Affects V.A.)

Shift 2 down (Affects H.A.)

➔ **V.A. at $x = -1$, H.A. at $y = -2$**

11. Since we know there is no V.A., the NPV (non-permissible value) is a P.D. So the unknown top factor is $(x - 3)$ *Same as bottom*

$$g(x) = \frac{3(x + 2)\cancel{(x - 3)}}{\cancel{(x - 3)}}$$

$$g(x) = 3(x + 2);\ x \neq 3$$

➔ $x = -2$ x-intercept, value of x that makes $x + 2$ equal 0.

x-coord of P.D. is $x = 3$. (Value of x that makes the top **and** bottom 0)

For y-coord, substitute $x = 3$ into the simplified equation form...

$$y = 3(3 + 2)$$

$$y = 3(5)$$

➔**Coords are $(3, 15)$**

12.

$$y = \frac{\cancel{x + 3}}{(x - 4)\cancel{(x + 3)}}$$

Here it's higher degree on bottom, so H.A. at $y = 0$

➔ (b) $y = 0$

$$y = \frac{1}{x - 4}, x \neq 3$$

V.A. at x value that makes the "bottom" (but not also the top) **0**

➔ (a) $x = 4$

x-coord of P.D. is $x = -3$. (Value of x that makes the top **and** bottom 0)

For y-coord, substitute $x = -3$ into the simplified equation form...

$$y = \frac{1}{-3 - 4}$$

➔ (c) **Coords are $\left(-3, -\frac{1}{7}\right)$**

13.

$$y = \frac{a(x - 0)}{(x + 1)}$$

x-int at $x = 0$ gives this factor on top

V.A. at $x = -1$ gives this factor on bottom

Vert. stretch, use any point on the graph to solve for this.

➔ $y = \dfrac{ax}{(x + 1)}$ Use pt $(1, -1)$ to solve for "a"

$$-1 = \frac{a(1)}{(1 + 1)}$$

$(-1)(2) = a(1)$ ➔ $a = -2$

Final Equation

$$y = \frac{-2x}{x + 1}$$

14. Vert. stretch, use any point on the graph to solve for this.

$\boldsymbol{b}$ is the x-coord of P.D. (P.D. gives factor on top and bottom)

$$y = \frac{a(x-b)}{(x-c)(x-b)}$$

$\boldsymbol{c}$ is the V.A. (V.A.s give factors on bottom only)

Note that we also know the bottom has two factors because the answer form is $x^2 + cx - d$

$$y = \frac{a(x-1)}{(x+4)(x-1)}$$

Use pt $(-3, 2)$ to solve for "a"

$$\mathbf{2} = \frac{a(\mathbf{-3}-1)}{(\mathbf{-3}+4)(\mathbf{-3}-1)}$$

$$\mathbf{2(1)(-4)} = (-4)a$$

$$\boldsymbol{a = -8/-4} \rightarrow \boldsymbol{a = 2}$$

So substituting "*a=2*" gives:

$$y = \frac{2(x-1)}{(x+4)(x-1)}$$

Expand bottom:

Final Equation

$$\boldsymbol{y = \frac{2(x-1)}{x^2+3x-4}}$$

15. Factor bottom to see NPVs / whether they are V.A.s or P.D.s.

$$y = \frac{a(x-b)(x-3)}{(2x+1)(x-3)}$$

$$y = \frac{a(x-b)\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}}$$

$$y = \frac{a(x-b)}{(2x+1)}$$

V.A. (from a factor that didn't cancel)

P.D. (from a factor that canceled)

ANSWER: B

16. x-intercept comes from value of x that makes top 0. (And that doesn't cancel with bottom – so used **simplified form**)

$x = b$

$$y = \frac{a(x-b)}{(2x+1)}$$

ANSWER: D

17. Here it's same degree top/ bottom, so H.A. at y =*ratio of lead coefficients*

→ **H.A. at** $y = \frac{\boldsymbol{a}(x-b)(x-3)}{\mathbf{2}x^2-5x-3}$

ANSWER: C

18. x-coord of P.D. is $= 3$, so factor on the bottom is $\mathbf{x-3}$ AND there must also be the same factor on top.

$$\boldsymbol{a = 3}$$

→ One of the factors on top is $x - 3$.

The remaining factor we can call $x - c$, giving us

$$y = \frac{(x-c)(x-3)}{x-3}, x \neq 3$$

Two approaches now… → **OR** (if you will)

$$x^2 - 5x + b = (x-c)(x-3)$$

The given "top"

Since one of the factors of the top is $x - 3$, **when** $\boldsymbol{x} =$ **3**, given top is **0**

$(3)^2 - 5(3) + b = 0$

$9 - 15 + b = 0$

$\boldsymbol{b = 6}$

ANSWER: 36

The remaining factor of the top is $\boldsymbol{x-c}$. And since there is a P.D. at $(3, 1)$, when we substitute $\boldsymbol{x = 3}$ into $\boldsymbol{x - c}$ we would get $\mathbf{1}$. (y-coord of PD)

$$\mathbf{(3)} - c = \mathbf{1}$$

$$c = \mathbf{2}$$

So, top is $(x-2)(x-3)$

Expands to $x^2 - 5x + 6$

$$\boldsymbol{b = 6}$$

19. We can a lot about the equation by just looking at the graph!

Since x-int at $a = 3$

$$y = \frac{a(x-3)(x-1)}{(x-4)(x-1)}$$

Since P.D.. at $x = 1$

Since V.A. at $x = 4$

Use the pt $(5, 4)$ to solve for "a"

$$\mathbf{4} = \frac{a(\mathbf{5}-3)\cancel{(\mathbf{5}-1)}}{(\mathbf{5}-4)\cancel{(\mathbf{5}-1)}}$$

$$\mathbf{4} = \frac{a\mathbf{(2)}}{\mathbf{(1)}} \Rightarrow \boldsymbol{a = 2}$$

So our equation becomes…

$$y = \frac{2(x-1)(x-3)}{(x-4)(x-1)}$$

…and expanding the bottom gives:

$$\boldsymbol{y = \frac{2(x-1)(x-3)}{x^2-5x+4}}$$

$\boldsymbol{a = 2}$ $\boldsymbol{b = 3}$ $\boldsymbol{c = 4}$

ANSWER: 234